13.1

$\phi\left(\mathbb{R}^2\right) = S^2 \setminus \left\{ \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$

$\phi(u,v) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + S\left(\begin{pmatrix} u \\ v \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right) \in S^2(1)$ とすると $\phi(u,v) = \begin{pmatrix} su \\ sv \\ 1-s \end{pmatrix}$ で $(su)^2 + (sv)^2 + (1-s)^2 = 1 \iff \left(u^2+v^2+1\right)s^2 - 2s = 0$ より、$s = \dfrac{2}{u^2+v^2+1}$ となる

$\psi^j = \phi \circ \partial_j^1 = \begin{pmatrix} \psi_1^j \\ \psi_2^j \\ \psi_3^j \end{pmatrix}$ とする

$\psi^0(t) = \begin{pmatrix} \dfrac{1-t}{t^2-t+1} \\ \dfrac{t}{t^2-t+1} \\ 1 - \dfrac{1}{t^2-t+1} \end{pmatrix}, \psi^1(t) = \begin{pmatrix} 0 \\ \dfrac{2t}{t^2+1} \\ 1 - \dfrac{2}{t^2+1} \end{pmatrix}, \psi^2(t) = \begin{pmatrix} \dfrac{2t}{t^2+1} \\ 0 \\ 1 - \dfrac{2}{t^2+1} \end{pmatrix}$ となる

$\mathrm{d}\psi_1^0 \dfrac{t^2-2t}{\left(t^2-t+1\right)^2}\mathrm{d}t, \mathrm{d}\psi_2^0 = -\dfrac{t^2-1}{\left(t^2-t+1\right)^2}\mathrm{d}t, \mathrm{d}\psi_3^0 = \dfrac{2t-1}{\left(t^2-t+1\right)^2}\mathrm{d}t$ より

$\left(\psi^0\right)^* \alpha = \left(\dfrac{1-2t}{\left(t^2-t+1\right)^3} + \left(\dfrac{1}{t^2-t+1} + 1 \right) \dfrac{2t-1}{\left(t^2-t+1\right)^2} \right) \mathrm{d}t = \dfrac{2t-1}{\left(t^2-t+1\right)^2}\mathrm{d}t$ なので

$$\begin{aligned} \int_{\phi \circ \partial_0^1 | \Delta^1} \alpha &= \int_0^1 \frac{2t-1}{\left(t^2-t+1\right)^2}\mathrm{d}t \\ &= \int \frac{\mathrm{d}}{\mathrm{d}t}\left(-\frac{1}{t^2-t+1} \right)\mathrm{d}t \\ &= 0 \end{aligned}$$

また $\mathrm{d}\psi_1^1 = 0, \mathrm{d}\psi_2^1 = \dfrac{2\left(1-t^2\right)}{\left(t^2+1\right)^2}\mathrm{d}t, \mathrm{d}\psi_3^1 = \dfrac{4t}{\left(t^2+1\right)^2}\mathrm{d}t$ より、$\left(\psi^1\right)^* \alpha = \dfrac{4(t+1)}{\left(t^2+1\right)^2}\mathrm{d}t$ なので、

$\displaystyle\int_{\phi \circ \partial_1^1 | \Delta^1} \omega = \int_0^1 \frac{4(t+1)}{\left(t^2+1\right)^2}\mathrm{d}t = \frac{\pi}{2}$

$\mathrm{d}\psi_1^2 = \dfrac{2\left(1-t^2\right)}{\left(t^2+1\right)^2}\mathrm{d}t, \mathrm{d}\psi_2^2 = 0, \mathrm{d}\psi_3^2 = \dfrac{4t}{\left(t^2+1\right)^2}\mathrm{d}t$ より $\left(\psi^2\right)^* \alpha = \dfrac{8t(t+1)}{\left(t^2+1\right)^3}\mathrm{d}t$ より

$$\begin{aligned} \int_{\phi \circ \partial_2^1 | \Delta^1} \omega &= \int_0^1 \frac{8t(t+1)}{\left(t^2+1\right)^3}\mathrm{d}t \\ &= \pi + 2 \end{aligned}$$

以上より、$\displaystyle\int_{\phi | \Delta^2} \mathrm{d}\omega = \frac{\pi}{2} + \frac{7}{2}$

13.2

$\psi^j = \phi \circ \partial_j^2 = \begin{pmatrix} \psi_1^j \\ \psi_2^j \\ \psi_3^j \\ \psi_4^j \end{pmatrix}$ とする

$\psi^0(t_1, t_2) = (1 - t_1 - t_2)\begin{pmatrix} \cos t_1 \\ \sin t_2 \\ \cos t_2 \\ \sin t_2 \end{pmatrix}, \psi^1(t_1, t_2) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

$\psi^2(t_1, t_2) = \begin{pmatrix} t_1 \\ 0 \\ t_1 \cos t_2 \\ t_1 \sin t_2 \end{pmatrix}, \psi^3(t_1, t_2) = \begin{pmatrix} t_1 \cos t_2 \\ t_1 \sin t_2 \\ t_1 \\ 0 \end{pmatrix}$ より、$\left(\psi^0\right)^* \omega = 0, \left(\psi^1\right)^* \omega = 0$ より、

$\int_{\phi \circ \partial_0^2 | \Delta^2} \omega = \int_{\phi \circ \partial_1^2 | \Delta^2} \omega = 0$

$\left(\psi^2\right)^* \omega = t_1^3 \mathrm{d}t_1 \wedge \mathrm{d}t_2, \left(\psi^3\right)^* \omega = t_1^3 \mathrm{d}t_1 \wedge \mathrm{d}t_2$

よって、$\int_{\phi \circ \partial_2^2 | \Delta^2} \omega = \int_{\phi \circ \partial_3^2 | \Delta^2} \omega = \iint_{\Delta^2} t_1^3 \mathrm{d}t_1 \mathrm{d}t_2$

以上より $\int_{\phi | \Delta^3} \mathrm{d}\omega = \int_{\phi \circ \partial_0^2 | \Delta^2} \omega - \int_{\phi \circ \partial_1^2 | \Delta^2} \omega + \int_{\phi \circ \partial_2^2 | \Delta^2} \omega - \int_{\phi \circ \partial_3^2 | \Delta^2} \omega$