§3

3.1

(1)

$\sigma_u = \begin{pmatrix} -r\sin u \\ r\cos u \\ 0 \end{pmatrix}, \sigma_v = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \sigma_u \times \sigma_v = \begin{pmatrix} r\cos u \\ r\sin u \\ 0 \end{pmatrix}$

$p\left(\frac{\pi}{3}, 1\right)$ を代入すると $\sigma_u \times \sigma_v = \begin{pmatrix} \frac{1}{2}r \\ \frac{\sqrt{3}}{2}r \\ 0 \end{pmatrix}$

よって、接平面の方程式は $\begin{pmatrix} x - \frac{1}{2}r \\ y - \frac{\sqrt{3}}{2}r \\ z - 1 \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{2}r \\ \frac{\sqrt{3}}{2}r \\ 0 \end{pmatrix} = 0$

(2)

$$\begin{aligned} \mathbf{n} &= \frac{\sigma_u \times \sigma_v}{\|\sigma_u \times \sigma_v\|} \\ &= \frac{1}{r}\begin{pmatrix} r\cos u \\ r\sin u \\ 0 \end{pmatrix} \end{aligned}$$

3.2

(1)

多項式は C^∞ から、σ も C^∞

$\sigma_u = \begin{pmatrix} 2u \\ 3u^2 - 2 \\ 0 \end{pmatrix}, \sigma_v = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \sigma_u \times \sigma_v = \begin{pmatrix} 3u^2 - 2 \\ -2u \\ 0 \end{pmatrix}$

$3u^2 - 2 = -2u = 0$ をみたす u は存在しないから、$\sigma_u \times \sigma_v \neq 0$

$(u, v), (u', v') \in D, \sigma(u, v) = \sigma(u', v')$ とする

$\Longrightarrow \begin{pmatrix} r\cos u \\ r\sin u \\ v \end{pmatrix} = \begin{pmatrix} r\cos u' \\ r\sin u' \\ v' \end{pmatrix} \Longrightarrow \begin{cases} v = v' \\ \cos u = \cos u' \\ \sin u = \sin u' \end{cases} \Longrightarrow \begin{cases} u = u' \\ v = v' \end{cases}$

(2)

p を代入すると、$\sigma_u \times \sigma_v = \begin{pmatrix} -2 \\ 0 \\ 0 \end{pmatrix}$

よって、接平面の方程式は $\begin{pmatrix} x + 2 \\ 0 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 0 \\ 0 \end{pmatrix} = 0$

(3)

$$\begin{aligned}\mathbf{n} &= \frac{\sigma_u \times \sigma_v}{\|\sigma_u \times \sigma_v\|} \\ &= \frac{1}{\sqrt{9u^4 - 8u^2 + 4}} \begin{pmatrix} 3u^2 - 2 \\ -2u \\ 0 \end{pmatrix}\end{aligned}$$

3.1

(1)

$\sigma_u = \begin{pmatrix} 1 \\ 0 \\ u \end{pmatrix}, \sigma_v = \begin{pmatrix} 0 \\ 1 \\ v \end{pmatrix}, \sigma_u \times \sigma_v = \begin{pmatrix} -u \\ -v \\ 1 \end{pmatrix}$

$\Pi : \begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = 0 \iff z = 0$

(2)

$\|\sigma_u \times \sigma_v\| = \sqrt{u^2 + v^2 + 1}$

$\mathbf{n} = \frac{\sigma_u \times \sigma_v}{\|\sigma_u \times \sigma_v\|} = \frac{1}{\sqrt{u^2 + v^2 + 1}} \begin{pmatrix} -u \\ -v \\ 1 \end{pmatrix}$

3.2

(1)

$\sigma_u = \begin{pmatrix} 6 \\ 0 \\ -2 \end{pmatrix}, \sigma_v = \begin{pmatrix} 0 \\ 6 \\ -3 \end{pmatrix}, \sigma_u \times \sigma_v = \begin{pmatrix} 12 \\ 18 \\ 36 \end{pmatrix}$

$\Pi : \begin{pmatrix} x - 6 \\ y - 6 \\ z + 4 \end{pmatrix} \cdot \begin{pmatrix} 12 \\ 18 \\ 36 \end{pmatrix} = 0 \iff 2(x - 6) + 3(y - 6) + 6(z + 4) = 0$

(2)

$\|\sigma_u \times \sigma_v\| = \sqrt{12^2 + 18^2 + 36^2} = 42$

$$\begin{aligned}\mathbf{n} &= \frac{\sigma_u \times \sigma_v}{\|\sigma_u \times \sigma_v\|} \\ &= \frac{1}{42} \begin{pmatrix} 12 \\ 18 \\ 36 \end{pmatrix} \\ &= \frac{1}{7} \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix}\end{aligned}$$

3.3

(1)

$$\sigma_u = \begin{pmatrix} 1 \\ 2u+2v \\ 3u^2+6uv \end{pmatrix}, \sigma_v = \begin{pmatrix} 0 \\ 2u \\ 3u^2 \end{pmatrix}, \sigma_u \times \sigma_v = \begin{pmatrix} -6u^2v \\ -3u^2 \\ 2u \end{pmatrix}$$

$$p = \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix} = \begin{pmatrix} u \\ u^2+2uv \\ u^3+3u^2v \end{pmatrix} \Longrightarrow \begin{cases} u=1 \\ v=-1 \end{cases}$$

$$\Pi : \begin{pmatrix} x-1 \\ y+1 \\ z+2 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ -3 \\ 2 \end{pmatrix} = 0 \iff 6\left(x-1\right) - 3\left(y+1\right) + 2\left(z+2\right) = 0$$

(2)

$$\|\sigma_u \times \sigma_v\| = \sqrt{36u^4v^2+9u^4+4u^2} = u\sqrt{9u^2\left(4v^2+1\right)+4}$$

$$\begin{aligned} \mathbf{n} &= \frac{\sigma_u \times \sigma_v}{\|\sigma_u \times \sigma_v\|} \\ &= \frac{1}{u\sqrt{9u^2\left(4v^2+1\right)+4}} \begin{pmatrix} -6u^2v \\ -3u^2 \\ 2u \end{pmatrix} \\ &= \frac{1}{\sqrt{9u^2\left(4v^2+1\right)+4}} \begin{pmatrix} -6uv \\ -3u \\ 2 \end{pmatrix} \end{aligned}$$