# 6.1

## (1)

$$f = x\sin\frac{z}{k} - y\cos\frac{z}{k}, \nabla f = \begin{pmatrix} \sin\frac{z}{k} \\ -\cos\frac{z}{k} \\ \frac{x}{k}\cos\frac{z}{k} + \frac{y}{k}\sin\frac{z}{k} \end{pmatrix}$$

$$T_pS = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 \middle| \begin{pmatrix} \sin\frac{z_0}{k} \\ -\cos\frac{z_0}{k} \\ \frac{x_0}{k}\cos\frac{z_0}{k} + \frac{y_0}{k}\sin\frac{z_0}{k} \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0 \right\}$$

$$\iff T_pS = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 \middle| x\sin\frac{z_0}{k} - y\cos\frac{z_0}{k} + \left(\frac{x_0}{k}\cos\frac{z_0}{k} + \frac{y_0}{k}\sin\frac{z_0}{k}\right) z = 0 \right\}$$

## (2)

$$\mathbf{n}\left(\sigma\left(u,v\right)\right) = \frac{\left(\nabla f\right)\left(\sigma\left(u,v\right)\right)}{\left\|\left(\nabla f\right)\sigma\left(u,v\right)\right\|} = \frac{1}{\sqrt{1+\frac{u^2}{k^2}}}\begin{pmatrix} \sin v \\ -\cos v \\ \frac{u}{k} \end{pmatrix} = \frac{k}{\sqrt{u^2+k^2}}\begin{pmatrix} \sin v \\ -\cos v \\ \frac{u}{k} \end{pmatrix}$$

$$\sigma_u = \begin{pmatrix} \cos v \\ \sin v \\ 0 \end{pmatrix}, \sigma_v = \begin{pmatrix} -u\sin v \\ u\cos v \\ k \end{pmatrix}$$

$$\begin{aligned} \sigma_u \times \sigma_v &= \begin{pmatrix} \cos v \\ \sin v \\ 0 \end{pmatrix} \times \begin{pmatrix} -u\sin v \\ u\cos v \\ k \end{pmatrix} \\ &= \begin{pmatrix} k\sin v \\ -k\cos v \\ u \end{pmatrix} \end{aligned}$$

$\|\sigma_u \times \sigma_v\| = \sqrt{u^2+k^2}$

だから、$\dfrac{\sigma_u \times \sigma_v}{\|\sigma_u \times \sigma_v\|} = \dfrac{1}{\sqrt{u^2+k^2}}\begin{pmatrix} k\sin v \\ -k\cos v \\ u \end{pmatrix} = \dfrac{k}{\sqrt{u^2+k^2}}\begin{pmatrix} \sin v \\ -\cos v \\ \frac{u}{k} \end{pmatrix}$

# 6.2

## (1)

$$f = x^2 - y^2 - r^2, \nabla f = \begin{pmatrix} 2x \\ -2y \\ 0 \end{pmatrix}$$

$$T_pS = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 \middle| \begin{pmatrix} 2x_0 \\ -2y_0 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0 \right\} = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 \middle| 2x_0x - 2y_0y = 0 \right\}$$

### (2)

$$\mathbf{n}\left(\sigma_+\left(u,v\right)\right)=\frac{\left(\nabla f\right)\left(\sigma_+\left(u,v\right)\right)}{\left\|\left(\nabla f\right)\left(\sigma_+\left(u,v\right)\right)\right\|}=\frac{1}{2r\sqrt{\cosh 2u}}\begin{pmatrix}2r\cosh u\\-2r\sinh u\\0\end{pmatrix}=\frac{1}{\sqrt{\cosh 2u}}\begin{pmatrix}\cosh u\\-\sinh u\\0\end{pmatrix}$$

$$\sigma_{+_u}=\begin{pmatrix}r\sinh u\\r\cosh u\\0\end{pmatrix},\sigma_{+_v}=\begin{pmatrix}0\\0\\1\end{pmatrix},\sigma_{+_u}\times\sigma_{+_v}=\begin{pmatrix}r\cosh u\\-r\sinh u\\0\end{pmatrix}$$

$$\frac{\sigma_{+_u}\times\sigma_{+_v}}{\left\|\sigma_{+_u}\times\sigma_{+_v}\right\|}=\frac{1}{\sqrt{\cosh 2u}}\begin{pmatrix}\cosh u\\-\sinh u\\0\end{pmatrix}=\mathbf{n}\left(\sigma_+\left(u,v\right)\right)$$

よって、$\sigma_+$ は正の向きである

$$\mathbf{n}\left(\sigma_-\left(u,v\right)\right)=\frac{\left(\nabla f\right)\left(\sigma_-\left(u,v\right)\right)}{\left\|\left(\nabla f\right)\left(\sigma_-\left(u,v\right)\right)\right\|}=\frac{1}{\sqrt{\cosh 2u}}\begin{pmatrix}-\cosh u\\-\sinh u\\0\end{pmatrix}$$

$$\sigma_{-_u}=\begin{pmatrix}-r\sinh u\\r\cosh u\\0\end{pmatrix},\sigma_{-_v}=\begin{pmatrix}0\\0\\1\end{pmatrix},\sigma_{-_u}\times\sigma_{-_v}=\begin{pmatrix}r\cosh u\\r\sinh u\\0\end{pmatrix}$$

$$\frac{\sigma_{-_u}\times\sigma_{-_v}}{\left\|\sigma_{-_u}\times\sigma_{-_v}\right\|}=\frac{1}{\sqrt{\cosh 2u}}\begin{pmatrix}\cosh u\\\sinh u\\0\end{pmatrix}=-\mathbf{n}\left(\sigma_-\left(u,v\right)\right)$$

だから、$\sigma_-$ は負の向き

## 6.1

### (1)

$$f=x^2+y^2-a^2\cosh^2\frac{z}{a},\nabla f=\begin{pmatrix}2x\\2y\\-a\sinh\dfrac{2z}{a}\end{pmatrix}$$

$$T_pS=\left\{\begin{pmatrix}x\\y\\z\end{pmatrix}\in\mathbb{R}^3\middle|\begin{pmatrix}2x_0\\2y_0\\-a\sinh\dfrac{2z_0}{a}\end{pmatrix}\cdot\begin{pmatrix}x\\y\\z\end{pmatrix}=0\right\}$$

$$\iff T_pS=\left\{\begin{pmatrix}x\\y\\z\end{pmatrix}\in\mathbb{R}^3\middle|2x_0x+2y_0y-az\sinh\frac{2z_0}{a}=0\right\}$$

### (2)

$$\mathbf{n}\left(\sigma\left(u,v\right)\right)=\frac{\left(\nabla f\right)\left(\sigma\left(u,v\right)\right)}{\left\|\left(\nabla f\right)\left(\sigma\left(u,v\right)\right)\right\|}=\frac{1}{2a\cosh^2 u}\begin{pmatrix}2\cosh u\cos v\\ 2\cosh u\sin v\\ -a\sinh 2u\end{pmatrix}$$

$$\sigma_u=\begin{pmatrix}a\sinh u\cos v\\ a\sinh u\sin v\\ a\end{pmatrix},\sigma_v=\begin{pmatrix}-a\cosh u\sin v\\ a\cosh u\cos v\\ 0\end{pmatrix}$$

$$\begin{aligned}\sigma_u\times\sigma_v&=\begin{pmatrix}a\sinh u\cos v\\ a\sinh u\sin v\\ a\end{pmatrix}\times\begin{pmatrix}-a\cosh u\sin v\\ a\cosh u\cos v\\ 0\end{pmatrix}\\&=\begin{pmatrix}-a^2\cosh u\cos v\\ -a^2\cosh u\sin v\\ a^2\sinh u\cosh u\end{pmatrix}\end{aligned}$$

$$\left\|\sigma_u\times\sigma_v\right\|=a^2\cosh^2 u$$

$$\frac{\sigma_u\times\sigma_v}{\left\|\sigma_u\times\sigma_v\right\|}=\frac{1}{a^2\cosh^2 u}\begin{pmatrix}-a^2\cosh u\cos v\\ -a^2\cosh u\sin v\\ a^2\sinh u\cosh u\end{pmatrix}=-\mathbf{n}\left(\sigma\left(u,v\right)\right)$$

よって、$\sigma$ は負の向き

## 6.2

### (1)

$$f=x^2+y^2-z^2-r^2,\nabla f=\begin{pmatrix}2x\\ 2y\\ -2z\end{pmatrix}$$

$$T_pS=\left\{\begin{pmatrix}x\\ y\\ z\end{pmatrix}\in\mathbb{R}^3\middle|\begin{pmatrix}2x_0\\ 2y_0\\ -2z_0\end{pmatrix}\cdot\begin{pmatrix}x\\ y\\ z\end{pmatrix}=0\right\}=\left\{\begin{pmatrix}x\\ y\\ z\end{pmatrix}\in\mathbb{R}^3\middle|x_0x+y_0y-z_0z=0\right\}$$

### (2)

$$\mathbf{n}\left(\sigma\left(u,v\right)\right)=\frac{\left(\nabla f\right)\left(\sigma\left(u,v\right)\right)}{\left\|\left(\nabla f\right)\left(\sigma\left(u,v\right)\right)\right\|}=\frac{1}{\sqrt{\cosh 2u}}\begin{pmatrix}\cosh u\cos v\\ \cosh u\sin v\\ -\sinh u\end{pmatrix}$$

$$\sigma_u=\begin{pmatrix}r\sinh u\cos v\\ r\sinh u\sin v\\ r\cosh u\end{pmatrix},\sigma_v=\begin{pmatrix}-r\cosh u\sin v\\ r\cosh u\cos v\\ 0\end{pmatrix},\sigma_u\times\sigma_v=\begin{pmatrix}-r^2\cosh^2 u\cos v\\ -r^2\cosh^2 u\sin v\\ r^2\sinh u\cosh u\end{pmatrix}$$

$$\frac{\sigma_u\times\sigma_v}{\left\|\sigma_u\times\sigma_v\right\|}=\frac{1}{r^2\cosh u\sqrt{\cosh 2u}}\begin{pmatrix}-r^2\cosh^2 u\cos v\\ -r^2\cosh^2 u\sin v\\ r^2\sinh u\cosh u\end{pmatrix}=\frac{1}{\sqrt{\cosh 2u}}\begin{pmatrix}-\cosh u\cos v\\ -\cosh u\sin v\\ \sinh u\end{pmatrix}$$

$$=-\mathbf{n}\left(\sigma\left(u,v\right)\right)$$

## 6.3

### (1)

$$f = x^2 + y^2 - z^2 + r^2, \nabla f = \begin{pmatrix} 2x \\ 2y \\ -2z \end{pmatrix}$$

$$T_pS = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 \middle| \begin{pmatrix} 2x_0 \\ 2y_0 \\ -2z_0 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0 \right\} = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 \middle| x_0x + y_0y - z_0z = 0 \right\}$$

### (2)

$$\mathbf{n}\left(\sigma_+\left(u, v\right)\right) = \frac{1}{\sqrt{r^2 + 2u^2 + 2v^2}} \begin{pmatrix} u \\ v \\ -\sqrt{r^2 + u^2 + v^2} \end{pmatrix}$$

$$\mathbf{n}\left(\sigma_-\left(u, v\right)\right) = \frac{1}{\sqrt{r^2 + 2u^2 + 2v^2}} \begin{pmatrix} u \\ v \\ \sqrt{r^2 + u^2 + v^2} \end{pmatrix}$$

$$\sigma_{+_u} = \begin{pmatrix} 1 \\ 0 \\ \frac{u}{\sqrt{r^2 + u^2 + v^2}} \end{pmatrix}, \sigma_{+_v} = \begin{pmatrix} 0 \\ 1 \\ \frac{v}{\sqrt{r^2 + u^2 + v^2}} \end{pmatrix}, \sigma_{+_u} \times \sigma_{+_v} = \begin{pmatrix} -\frac{u}{\sqrt{r^2+u^2+v^2}} \\ -\frac{v}{\sqrt{r^2+u^2+v^2}} \\ 1 \end{pmatrix}$$

$$\sigma_{-_u} = \begin{pmatrix} 1 \\ 0 \\ -\frac{u}{\sqrt{r^2 + u^2 + v^2}} \end{pmatrix}, \sigma_{-_v} = \begin{pmatrix} 0 \\ 1 \\ -\frac{v}{\sqrt{r^2 + u^2 + v^2}} \end{pmatrix}, \sigma_{-_u} \times \sigma_{-_v} = \begin{pmatrix} \frac{u}{\sqrt{r^2+u^2+v^2}} \\ \frac{v}{\sqrt{r^2+u^2+v^2}} \\ 1 \end{pmatrix}$$

$$\frac{\sigma_{+_u} \times \sigma_{+_v}}{\|\sigma_{+_u} \times \sigma_{+_v}\|} = \frac{1}{\sqrt{r^2 + 2u^2 + 2v^2}} \begin{pmatrix} -u \\ -v \\ \sqrt{r^2 + u^2 + v^2} \end{pmatrix} = -\mathbf{n}\left(\sigma_+\left(u, v\right)\right)$$

$$\frac{\sigma_{-_u} \times \sigma_{-_v}}{\|\sigma_{-_u} \times \sigma_{-_v}\|} = \frac{1}{\sqrt{r^2 + 2u^2 + 2v^2}} \begin{pmatrix} u \\ v \\ \sqrt{r^2 + u^2 + v^2} \end{pmatrix} = \mathbf{n}\left(\sigma_-\left(u, v\right)\right)$$

よって、正になるものは $\sigma_-$