§3
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K3

(1)

(a)

$$
\begin{aligned}
&(\mathbf{c_1}+\mathbf{c_2})' \times (\mathbf{c_1}-\mathbf{c_2})' \\
&= \left(\mathbf{c_1}'+\mathbf{c_2}'\right) \times \left(\mathbf{c_1}'-\mathbf{c_2}'\right) \\
&= ((-2\sin 2t, 2\cos 2t, 0)+(2t, 2t-2, 2t+2)) \\
&\times ((-2\sin 2t, 2\cos 2t, 0)-(2t, 2t-2, 2t+2)) \\
&= (2t-2\sin 2t, 2t+2\cos 2t-2, 2) \times (-2t-2\sin 2t, 2\cos 2t-2t+2, -2t-2) \\
&= \begin{pmatrix} \begin{vmatrix} 2t+2\cos 2t-2 & 2\cos 2t-2t+2 \\ 2 & -2t-2 \end{vmatrix} \\ \begin{vmatrix} 2 & -2t-2 \\ 2t-2\sin 2t & -2t-2\sin 2t \end{vmatrix} \\ \begin{vmatrix} 2t-2\sin 2t & -2t-2\sin 2t \\ 2t+2\cos 2t-2 & 2\cos 2t-2t+2 \end{vmatrix} \end{pmatrix} \\
&= \begin{pmatrix} (2t+2\cos 2t-2)(-2t-2)-2(2\cos 2t-2t+2) \\ 2(-2t-2\sin 2t)+(2t+2)(2t-2\sin 2t) \\ (2t-2\sin 2t)(2\cos 2t-2t+2)+(2t+2\sin 2t)(2t+2\cos 2t-2) \end{pmatrix} \\
&= \begin{pmatrix} -4t^2+4t-4t\cos 2t-8\cos 2t \\ 4t^2-4t\sin 2t-8\sin 2t \\ 8t\sin 2t-8\sin 2t+8t\cos 2t \end{pmatrix}
\end{aligned}
$$

(b)

$$\begin{aligned}
&\left(\mathbf{c_2} \times \mathbf{c_2'}\right)' \\
&= \mathbf{c_2'} \times \mathbf{c_2'} + \mathbf{c_2} \times \mathbf{c_2''} \\
&= \left(t^2+2, t^2-2t, t^2+2t\right) \times (2,2,2) \\
&= \left(\begin{array}{c} \left|\begin{array}{cc} t^2-2t & 2 \\ t^2+2t & 2 \end{array}\right| \\ \left|\begin{array}{cc} t^2+2t & 2 \\ t^2+2 & 2 \end{array}\right| \\ \left|\begin{array}{cc} t^2+2 & 2 \\ t^2-2t & 2 \end{array}\right| \end{array}\right) \\
&= \left(\begin{array}{c} -8t \\ 4t-4 \\ 4t+4 \end{array}\right)
\end{aligned}$$

(2)

(a)

$$\begin{aligned}
\frac{\partial}{\partial u_i}(X \cdot Y) &= \sum_{k=1}^{m}\left(\frac{\partial X_k}{\partial u_i} \cdot Y_k + X_k \cdot \frac{\partial Y_k}{\partial u_i}\right) \\
&= \sum_{k=1}^{m} \frac{\partial X_k}{\partial u_i} \cdot Y_k + \sum_{k=1}^{m} X_k \cdot \frac{\partial Y_k}{\partial u_i} \\
&= \frac{\partial X}{\partial u_i} \cdot Y + X \cdot \frac{\partial Y}{\partial u_i}
\end{aligned}$$

(b)

$\dfrac{\partial X}{\partial u_i}(u)$と$X(u)$は直交する

$\Longleftrightarrow \dfrac{\partial X(u)}{\partial u_i} \cdot X(u) = 0$

ここで$X(u) \in \{x \in \mathbb{R}^n \mid |x| = C\}$から、一回偏微分すると成分は0になるため、$X(u)$との内積は必ず0である

言い換えれば、$\dfrac{\partial X}{\partial u_i}(u)$と$X(u)$は直交している

P3.1

(⇒)
$\lim_{t \to t_0} c(t) = v$より
$\forall \epsilon > 0, \exists \delta > 0, s.t. |t - t_0| < \delta \Rightarrow |c(t_0) - v| < \epsilon$
i.e.$\forall i \in \{1, 2, \ldots, n\}, \forall \epsilon > 0, \exists \delta > 0, s.t. |t - t_0| < \delta \Rightarrow |c_i(t_0) - v_i| < \epsilon$
(⇐)
$\lim_{t \to t_0} c_i(t) = v_i$より、
$\forall i \in \{1, 2, \ldots, n\}, \forall \epsilon_i > 0, \exists \delta > 0, s.t. |t - t_0| < \delta \Rightarrow |c_i(t) - v_i| < \epsilon_i$
そして、$\epsilon := \min\{\epsilon_i\}$とすれば、$\exists \delta > 0, s.t. |t - t_0| < \delta \Rightarrow |c(t) - v| < \epsilon$

P3.2

3.1より、c_1とc_2の各成分がそれぞれv_{1_i}, v_{2_i}に収束する、そうなると、$c_1 + c_2$の各成分$c_{1_i} + c_{2_i}$も$v_{1_i} + v_{2_i}$に収束する
言い換えれば、$c_1 + c_2$も$v_1 + v_2$に収束する(⇒の右側)

P3.3

$|f(t)c(t) - av| \leq |f(t)||c(t) - v| + |f(t) - a||v|$を注意すると実数の極限と同じように証明できる

P3.4

$c(t)$はベクトル値関数だから、各成分に対して、普通の合成関数の微分を使うと、$\forall i, \frac{\mathrm{d}}{\mathrm{d}s} c_i(t(s)) = \frac{\mathrm{d}c_i(t(s_0))}{\mathrm{d}t} \cdot \frac{\mathrm{d}t(s_0)}{\mathrm{d}s}$が成り立つ、そうすると、$i$の任意性より、$\frac{\mathrm{d}}{\mathrm{d}s} c(t(s)) = \frac{\mathrm{d}c(t(s_0))}{\mathrm{d}t} \cdot \frac{\mathrm{d}t(s_0)}{\mathrm{d}s}$

P3.5

$$
\begin{aligned}
\text{LHS} &= \frac{\partial}{\partial u_i}\left(\begin{vmatrix} X_2 & Y_2 \\ X_3 & Y_3 \end{vmatrix}, \begin{vmatrix} X_3 & Y_3 \\ X_1 & Y_1 \end{vmatrix}, \begin{vmatrix} X_1 & Y_1 \\ X_2 & Y_2 \end{vmatrix}\right) \\
&= \begin{pmatrix}
\begin{vmatrix} \frac{\partial}{\partial u_i} X_2 & Y_2 \\ \frac{\partial}{\partial u_i} X_3 & Y_3 \end{vmatrix} + \begin{vmatrix} X_2 & \frac{\partial}{\partial u_i} Y_2 \\ X_3 & \frac{\partial}{\partial u_i} Y_3 \end{vmatrix} \\
\begin{vmatrix} \frac{\partial}{\partial u_i} X_3 & Y_3 \\ \frac{\partial}{\partial u_i} X_1 & Y_1 \end{vmatrix} + \begin{vmatrix} X_3 & \frac{\partial}{\partial u_i} Y_3 \\ X_1 & \frac{\partial}{\partial u_i} Y_1 \end{vmatrix} \\
\begin{vmatrix} \frac{\partial}{\partial u_i} X_1 & Y_1 \\ \frac{\partial}{\partial u_i} X_2 & Y_2 \end{vmatrix} + \begin{vmatrix} X_1 & \frac{\partial}{\partial u_i} Y_1 \\ X_2 & \frac{\partial}{\partial u_i} Y_2 \end{vmatrix}
\end{pmatrix} \\
&= \text{RHS}
\end{aligned}
$$